International Baccalaureate
Baccalauréat International
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Diploma Programme

Mathematics: analysis and approaches SL formula booklet

For use during the course and in the examinations
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Version 1.0

STANDARD LEVEL

Contents

Topic 1: Number and algebra – SL

1.2	The nth term of an arithmetic sequence The sum of n terms of an arithmetic sequence	$u_n = u_1 + (n-1)d$ $S_n = \frac{n}{2}\left(2u_1 + (n-1)d\right)$; $S_n = \frac{n}{2}(u_1 + u_n)$
1.3	The nth term of a geometric sequence The sum of n terms of a finite geometric sequence	$u_n = u_1 r^{n-1}$ $S_n = \frac{u_1(r^n - 1)}{r-1} = \frac{u_1(1-r^n)}{1-r}$, $r \neq 1$
1.8	The sum of an infinite geometric sequence	$S_\infty = \frac{u_1}{1-r}$, $\lvert r \rvert < 1$
1.4	Compound interest	$FV = PV \times \left(1 + \frac{r}{100k}\right)^{kn}$, where FV is the future value, PV is the present value, n is the number of years, k is the number of compounding periods per year, r% is the nominal annual rate of interest
1.5	Exponents and logarithms	$a^x = b \Leftrightarrow x = \log_a b$, where $a > 0, b > 0, a \neq 1$
1.7	Exponents and logarithms Exponential and logarithmic functions	$\log_a xy = \log_a x + \log_a y$ $\log_a \frac{x}{y} = \log_a x - \log_a y$ $\log_a x^m = m \log_a x$ $\log_a x = \frac{\log_b x}{\log_b a}$ $a^x = e^{x \ln a}$; $\log_a a^x = x = a^{\log_a x}$ where $a, x > 0, a \neq 1$
1.9	Binomial theorem $n \in \mathbb{N}$	$(a+b)^n = a^n + {}^nC_1\, a^{n-1}b + \ldots + {}^nC_r\, a^{n-r}b^r + \ldots + b^n$ ${}^nC_r = \frac{n!}{r!(n-r)!}$

Topic 2: Functions – SL

2.1	Equations of a straight line Gradient formula	$y = mx + c$; $ax + by + d = 0$; $y - y_1 = m(x - x_1)$ $m = \dfrac{y_2 - y_1}{x_2 - x_1}$
2.6	Axis of symmetry of the graph of a quadratic function	$f(x) = ax^2 + bx + c \Rightarrow$ axis of symmetry is $x = -\dfrac{b}{2a}$
2.7	Solutions of a quadratic equation Discriminant	$ax^2 + bx + c = 0 \Rightarrow x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a},\ a \neq 0$ $\Delta = b^2 - 4ac$

Topic 3: Geometry and trigonometry – SL

Prior learning – SL	
Area of a parallelogram	$A = bh$, where b is the base, h is the height
Area of a triangle	$A = \frac{1}{2}(bh)$, where b is the base, h is the height
Area of a trapezoid	$A = \frac{1}{2}(a+b)h$, where a and b are the parallel sides, h is the height
Area of a circle	$A = \pi r^2$, where r is the radius
Circumference of a circle	$C = 2\pi r$, where r is the radius
Volume of a cuboid	$V = lwh$, where l is the length, w is the width, h is the height
Volume of a cylinder	$V = \pi r^2 h$, where r is the radius, h is the height
Volume of a prism	$V = Ah$, where A is the area of cross-section, h is the height
Area of the curved surface of a cylinder	$A = 2\pi rh$, where r is the radius, h is the height
Distance between two points (x_1, y_1) and (x_2, y_2)	$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$
Coordinates of the midpoint of a line segment with endpoints (x_1, y_1) and (x_2, y_2)	$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

3.1	Distance between two points (x_1, y_1, z_1) and (x_2, y_2, z_2)	$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$
	Coordinates of the midpoint of a line segment with endpoints (x_1, y_1, z_1) and (x_2, y_2, z_2)	$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$

	Volume of a right-pyramid	$V = \frac{1}{3}Ah$, where A is the area of the base, h is the height
	Volume of a right cone	$V = \frac{1}{3}\pi r^2 h$, where r is the radius, h is the height
	Area of the curved surface of a cone	$A = \pi r l$, where r is the radius, l is the slant height
	Volume of a sphere	$V = \frac{4}{3}\pi r^3$, where r is the radius
	Surface area of a sphere	$A = 4\pi r^2$, where r is the radius
3.2	Sine rule	$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$
	Cosine rule	$c^2 = a^2 + b^2 - 2ab\cos C$; $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$
	Area of a triangle	$A = \frac{1}{2}ab\sin C$
3.4	Length of an arc	$l = r\theta$, where r is the radius, θ is the angle measured in radians
	Area of a sector	$A = \frac{1}{2}r^2\theta$, where r is the radius, θ is the angle measured in radians
3.5	Identity for $\tan\theta$	$\tan\theta = \frac{\sin\theta}{\cos\theta}$
3.6	Pythagorean identity	$\cos^2\theta + \sin^2\theta = 1$
	Double angle identities	$\sin 2\theta = 2\sin\theta\cos\theta$ $\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$

Topic 4: Statistics and probability – SL

4.2	Interquartile range	$\text{IQR} = Q_3 - Q_1$
4.3	Mean, $\bar{x}$, of a set of data	$\bar{x} = \dfrac{\sum_{i=1}^{k} f_i x_i}{n}$, where $n = \sum_{i=1}^{k} f_i$
4.5	Probability of an event A	$\mathrm{P}(A) = \dfrac{n(A)}{n(U)}$
	Complementary events	$\mathrm{P}(A) + \mathrm{P}(A') = 1$
4.6	Combined events	$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$
	Mutually exclusive events	$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B)$
	Conditional probability	$\mathrm{P}(A \mid B) = \dfrac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$
	Independent events	$\mathrm{P}(A \cap B) = \mathrm{P}(A)\,\mathrm{P}(B)$
4.7	Expected value of a discrete random variable X	$\mathrm{E}(X) = \sum_{i=1}^{k} x_i \, \mathrm{P}(X = x_i)$
4.8	Binomial distribution $X \sim \mathrm{B}(n, p)$	
	Mean	$\mathrm{E}(X) = np$
	Variance	$\mathrm{Var}(X) = np(1-p)$
4.12	Standardized normal variable	$z = \dfrac{x - \mu}{\sigma}$

Topic 5: Calculus – SL

5.3	Derivative of x^n	$f(x)=x^n \Rightarrow f'(x)=nx^{n-1}$
5.6	Derivative of $\sin x$	$f(x)=\sin x \Rightarrow f'(x)=\cos x$
	Derivative of $\cos x$	$f(x)=\cos x \Rightarrow f'(x)=-\sin x$
	Derivative of e^x	$f(x)=\mathrm{e}^x \Rightarrow f'(x)=\mathrm{e}^x$
	Derivative of $\ln x$	$f(x)=\ln x \Rightarrow f'(x)=\frac{1}{x}$
	Chain rule	$y=g(u)$, where $u=f(x) \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\mathrm{d}y}{\mathrm{d}u}\times\frac{\mathrm{d}u}{\mathrm{d}x}$
	Product rule	$y=uv \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=u\frac{\mathrm{d}v}{\mathrm{d}x}+v\frac{\mathrm{d}u}{\mathrm{d}x}$
	Quotient rule	$y=\frac{u}{v} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=\frac{v\frac{\mathrm{d}u}{\mathrm{d}x}-u\frac{\mathrm{d}v}{\mathrm{d}x}}{v^2}$
5.9	Acceleration	$a=\frac{\mathrm{d}v}{\mathrm{d}t}=\frac{\mathrm{d}^2s}{\mathrm{d}t^2}$
	Distance travelled from t_1 to t_2	distance $=\int_{t_1}^{t_2}\lvert v(t)\rvert\,\mathrm{d}t$
	Displacement from t_1 to t_2	displacement $=\int_{t_1}^{t_2} v(t)\,\mathrm{d}t$
5.5	Integral of x^n	$\int x^n\,\mathrm{d}x=\frac{x^{n+1}}{n+1}+C,\ n\neq -1$
	Area between a curve $y=f(x)$ and the x-axis, where $f(x)>0$	$A=\int_a^b y\,\mathrm{d}x$

5.10	Standard integrals	$\int \frac{1}{x}\,\mathrm{d}x = \ln\lvert x\rvert + C$ $\int \sin x\,\mathrm{d}x = -\cos x + C$ $\int \cos x\,\mathrm{d}x = \sin x + C$ $\int \mathrm{e}^{x}\,\mathrm{d}x = \mathrm{e}^{x} + C$
5.11	Area of region enclosed by a curve and x-axis	$A = \int_{a}^{b} \lvert y \rvert\,\mathrm{d}x$